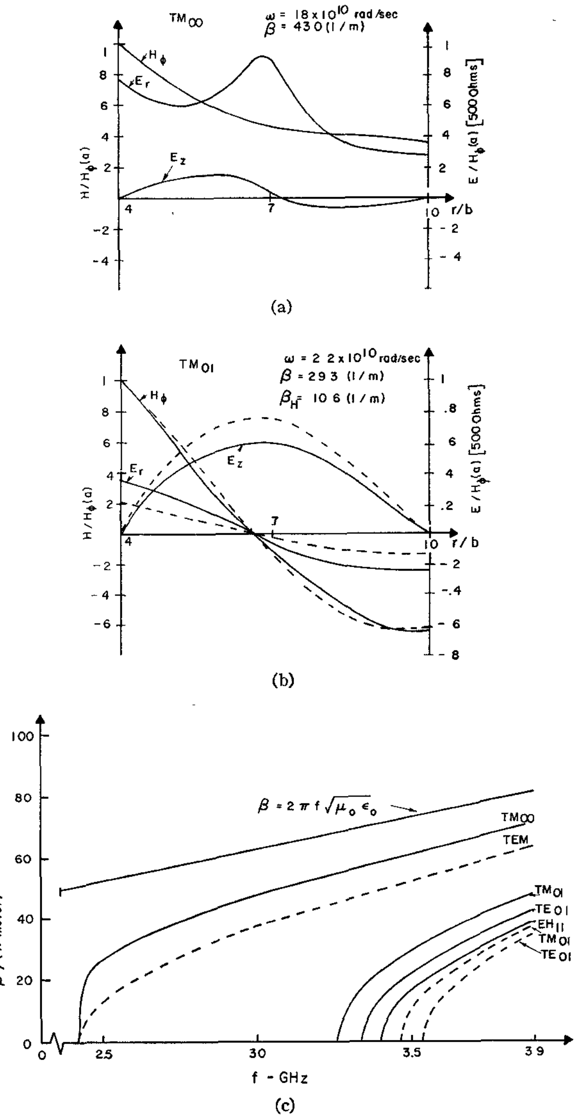

Fig. 4. Typical EM field distributions for permittivity of Example 2. (a) TM_{00} mode (fundamental mode). (b) TM_{01} mode (hashed curves represent field distributions in a coaxial line filled uniformly with a plasma having $\Pi = 15.25 \times 10^9$ rad/s; β_H is the propagation constant for the uniform case.) (c) Dispersion curves for permittivity of Example 2 and for a coaxial waveguide filled with a homogeneous plasma.

fields that would exist in a coaxial line, if the electron plasma frequency were uniform and equal to Π. It is seen that the alterations in the field components for the higher order modes are smaller because the permittivity inhomogeneity is not appreciable in the frequency range where the higher modes propagate.

The field variations for the first two hybrid modes are shown in Fig. 3 when $\omega/\Pi = 2.29$. These fields were obtained by systematically trying different ratios of $H_\phi(a)/H_z(a)$ in the search procedure outlined in Section II. In the case of the EH_{11} mode, this ratio is approximately $H_\phi(a)/H_z(a) = 0.176$, and for the EH_{12} mode, $H_\phi(a)/H_z(a) = 0.230$. The cutoff frequencies of higher hybrid modes are sufficiently great that the permittivity of the medium is essentially that of free space in the frequency range where these modes propagate. Consequently, these modes are approximately linear combinations of TE_{mn} and TM_{mn} modes that can propagate in a homogeneous coaxial line.

Dispersion curves (solid lines) for the modes discussed above are seen in Fig. 4(c) for the example treated. Dispersion plots (dotted lines) are also given for those modes that would propagate along a homogeneously filled plasma coaxial line. Although the cutoff frequency of the fundamental (TM_{00}) mode occurs at the same frequency ($\omega = \Pi$) as the cutoff frequency of the homogeneous line, the cutoff appears to be much sharper. The cutoff frequencies for the higher order modes, on the other hand, differ significantly. For these modes, the cutoff frequency obtained by considering the dielectric inhomogeneity is lower than that of coaxial modes in a homogeneous line having a dielectric constant $\epsilon = \epsilon_0(1 - \Pi^2/\omega^2)$. The cutoff frequencies of these inhomogeneous modes correspond more closely to the cutoff frequency of modes in a uniform coaxial line where the dielectric constant is the average value

$$\langle \epsilon \rangle = \left[\int_a^b \epsilon(r) r \, dr \right] / (b - a).$$

C. Discussion

The speed and accuracy with which the field calculations were carried out indicate that the procedure employed is practical for arbitrary radial inhomogeneities of the electrical properties of the medium. In every instance, the computing time required to calculate the field solution at each frequency for the above examples was less than 10 s. The accuracy of these solutions was checked by substituting the numerical solutions obtained into Maxwell's divergence equations, which for our system can be written as

$$H_r' = -\frac{n}{r} H_\phi + \beta H_z - \frac{H_r}{r} \tag{10}$$

and

$$E_r' = \frac{nE_\phi}{r} - \beta E_z - \left[\frac{1}{r} - \frac{\epsilon'}{\epsilon}\right] E_r. \tag{11}$$

In every case, the values of H_r' and E_r' obtained from (10) and (11) were within 1 percent of the values calculated in the main routine. The accuracy of the procedure was also checked by comparing numerical solutions with known analytical solutions of the higher order modes in a homogeneous coaxial line. The computed cutoff frequencies and dispersion curves were within 1 percent of the analytically determined values below the $TE_{0,30}$ coaxial waveguide mode.

References

[1] E. Ah Sam and V. Klinger, "Propagation in cylindrical waveguide containing inhomogeneous dielectric," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-15, pp. 60–61, Jan. 1967.
[2] R. Yamada and K. Watanabe, "Propagation in cylindrical waveguide containing inhomogeneour dielectric," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol MTT-13, pp. 716–717, Sept. 1965.
[3] G. Goubau, "Surface waves and their application to transmission lines," *J. Appl. Phys.*, vol. 21, pp. 1119–1128, Nov. 1950.
[4] J. L. Shohet and A. J. Hatch, "Eigenvalues of a microwave cavity filled with a plasma of variable radial density," *J. Appl. Phys.*, vol. 41, pp. 2610–2618, May 1970.
[5] V. N. Krepak and I. P. Yakimenko, "E-waves in circular waveguide with inhomogeneous plasma," *Radio Eng. Electron. Phys.*, vol. 14, no. 3, pp. 364–369, Mar. 1969.
[6] W. J. English, "Vector variational solutions of inhomogeneously loaded cylindrical waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 9–18, Jan. 1971.
[7] B. Carnanan, H. A. Luther, and J. O. Wilkes, *Applied Numerical Methods*. New York: Wiley, 1969, pp. 405–406.
[8] R. E. McIntosh and R. G. Andrews, "Dispersion of transient electromagnetic signals propagating along a coaxial gas discharge," *J. Appl. Phys.*, vol. 42, no. 5, pp. 1857–1862, Apr. 1971.

A Quick Accurate Method to Measure the Dielectric Constant of Microwave Integrated-Circuit Substrates

JOHN Q. HOWELL

***Abstract*—A technique is described that makes possible the accurate measurement of the dielectric constant of microwave integrated-circuit substrates. The substrate is metallized on all sides, hence forming a tiny resonant cavity, and the resonant frequencies are determined either from transmission or reflection. The dielectric constant is then calculated to an accuracy of better than 1 percent.**

Manuscript received June 1, 1972; revised July 13, 1972.
The author is with the NASA Langley Research Center, Hampton, Va. 23365.

Introduction

Before microstrip circuits can be accurately designed, it is necessary to know the dielectric constant of the substrate material being used. This is particularly important when using alumina because of the wide variations encountered among different venders and even different orders from the same vender [1]. The technique described by Lenzing in [1] has the disadvantage that it requires specially prepared substrates and a specially constructed substrate holder. While this technique is very accurate, there is still need of a method that can be used in the laboratory to quickly measure the dielectric constant of a substrate upon which a microstrip circuit will be constructed. Napoli and Hughes [2] gave such a method, but its accuracy is questionable due to radiation losses. This short paper presents a variation of the Napoli–Hughes technique that is more accurate while still being easily and quickly implemented.

Technique

This technique, like Lenzing's, uses a substrate metallized on all sides so that a small microwave cavity is formed. Solving the cavity resonant-frequency equation for the dielectric constant one obtains

$$\epsilon = \frac{c^2}{4f_{pq}^2}\left\{\frac{p^2}{a^2} + \frac{q^2}{b^2}\right\} \tag{1}$$

where ϵ is the dielectric constant, c is the speed of light, a and b are the substrate dimensions, and f_{pq} is the cavity resonant frequency for the (p, q) mode. Napoli and Hughes suggested detecting the resonant frequencies by using either a network analyzer or a sweep generator and a crystal detector to measure the energy transmitted through the cavity as a function of frequency. Regardless of the equipment used the corners of the cavity should be cleared of the metallization and inserted in the ends of the APC-7 connectors as they suggested. It was also found that either male- or female-type N connectors could be similarly used. Peaks in the curve of transmission versus frequency as shown in Fig. 1 correspond to the resonant frequencies of the cavity, and from (1) the dielectric constant can be computed. Table I gives the measured and calculated data obtained for the two cases shown in Fig. 1. It is also possible to detect dips in the reflection from the cavity, but the transmission peaks are somewhat easier to observe.

Discussion

Referring to Fig. 1 and Table I a comparison can be made between the Napoli–Hughes technique and the method suggested in this short paper. Note that when using their technique the resulting Q's were much lower, indicating that the edges were radiating instead of being open circuits as they assumed. Hence the resonant frequencies are shifted and the calculated dielectric constants are lower and not as self-consistent as the results obtained using the method described here.

The substrate used to make the above comparison had open edges. The closed-edge measurements were made by painting the edges with silver conducting paint. It is very important to apply enough paint so that the conducting surface is several skin depths thick. Coupling holes were then provided by scraping the edges at opposite corners. It is also helpful to use tinfoil or small strips of metal to block alternate transmission paths on the outside surface of the cavity.

From (1) it is seen that the two sources of error are the frequency and length measurements. If the length is accurate to 0.1 percent, then it contributes 0.2-percent error to the dielectric constant. If a counter is used to measure frequency, the error in the resonant frequency will be mainly that caused by wall losses. If the Q is measured, the following correction can be made [3]:

$$f_0 = f_m \bigg/ \left(1 - \frac{1}{2Q}\right) \tag{2}$$

where f_m is the measured frequency and f_0 is the corrected frequency that should be used in (1) above. If the correction given by (2) is not made, the method described in this short paper results in error from this source of less than 0.4 percent. Hence, unless extremely accurate results are desired, it is not necessary to measure the Q of each resonance and apply (2) to correct the frequency.

In conclusion a simple technique is described that allows the microwave engineer to accurately and quickly measure the dielectric constant of a substrate that is later used for a microstrip circuit. This technique applies to any substrate material but is particularly useful when applied to alumina because of the wide variations of dielectric constants that are observed.

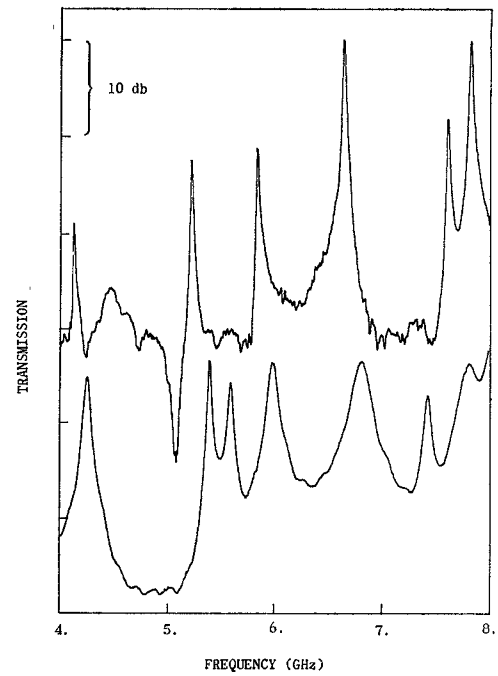

Fig. 1. Upper curve is the closed-edge cavity transmission using the technique suggested in the paper. Lower curve is the open-edge cavity transmission using the Napoli–Hughes technique. The same alumina substrate (0.994 in by 0.994 in by 0.025 in) was used for both curves.

TABLE I

Comparison of Q and Dielectric Constant for Open- and Closed-Edge Substrate Resonators

Mode Numbers (p,q)	Resonant Frequency GHz	Q	Dielectric Constant
Closed-Edge Substrate Resonator			
(2,1)	4.124	258	10.36
(2,2)	5.218	290	10.36
(3,1)	5.836	292	10.35
(2,3)	6.651	332	10.36
(1,4)	7.613	346	10.34
(3,3)	7.830	326	10.35
Open-Edged Substrate Resonator (Napoli–Hughes)			
(2,1)	4.230	88	9.85
(2,2)	5.392	154	9.70
(3,0)	5.576	153	10.20
(3,1)	5.972	75	9.88
(3,2)	6.803	63	9.90
(4,0)	7.438	114	10.19
(4,1)	7.768	55	9.93

References

[1] H. F. Lenzing, "Measurement of dielectric constant of ceramic substrates at microwave frequencies," presented at the Electronics Division, American Ceramic Society Meeting, Washington, D. C., May 10, 1972, Paper 41.
[2] L. S. Napoli and J. J. Hughes, "A simple technique for the accurate determination of the microwave dielectric constant for microwave integrated circuit substrates," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 664–665, July 1971.
[3] R. E. Collin, *Foundations for Microwave Engineering.* New York: McGraw-Hill, 1966, pp. 354–355.

Dispersion of Parallel-Coupled Microstrip

W. J. GETSINGER

***Abstract*—Dispersion predictions for the even and odd modes of parallel-coupled microstrip obtained by using the LSE-mode model for microstrip are found to be in good agreement with recently published measurements. Both the basic dispersion relationship and the empirical factor *G* appear to be the same as for single microstrip lines if the even- and odd-mode impedances used are those of the total parallel-coupled configuration rather than those of a single line of the coupled pair.**

Introduction

Measured dispersion data on the even and odd modes of coupled microstrip lines have recently been published by Gould and Talboys [1], who measured the resonances of rings of concentric pairs of microstrip lines on an alumina substrate 0.025 in thick. These data will be used to verify the applicability of the LSE-mode model [2] of microstrip propagation to coupled microstrip lines.

Dispersion Analysis

A summary of the parameters of the lines described in [1] is given in Table I. Physical dimensions of the coupled lines are shown in Fig. 1. From [2] the dispersion relationships are

$$\epsilon_e = \epsilon_s - \frac{\epsilon_s - \epsilon_{e0}}{1 + G(f^2/f_p^2)} \tag{1}$$

$$f_p = \frac{Z_0}{2\mu H} \tag{2}$$

$$G \approx 0.6 + 0.009 Z_0 \tag{3}$$

where

- ϵ_e the frequency-sensitive effective dielectric constant;
- ϵ_{e0} the effective dielectric constant at zero frequency;
- ϵ_s the substrate dielectric constant;
- Z_0 the characteristic impedance at zero frequency;
- μ 31.92 nH/in;
- G an empirical parameter;
- H the substrate thickness.

In literature on the design of microwave components with parallel-coupled lines, it is conventional to define even-mode impedance Z_{0e} and odd-mode impedance Z_{0o} with reference to a single one of the coupled lines measured to ground, although the actual modes travel on the pair of lines.

In the even mode the two strips are at the same potential, and the total current is twice that on a single strip. Thus the total mode impedance is half that of a single strip, and dispersion for even-mode propagation is computed by substituting $Z_{0e}/2$ for Z_0 in (2) and (3).

In the odd mode the two strips are at opposite potentials, and the voltage between strips is twice that of a single strip to ground. Thus the total mode impedance is twice that of a single strip, and the dispersion for odd-mode propagation is computed by substituting $2Z_{0o}$ for Z_0 in (2) and (3).

The plotted points of guide wavelength presented by Gould and Talboys [1] for the coupled microstrips of Table I were used to compute values of effective dielectric constant. These values are shown by the circles in Fig. 2. For each set of points, a value of effective dielectric constant at zero frequency ϵ_{e0} was estimated, consistent with the requirement that the slope of the dispersion curve must be zero at zero frequency. Values from Table I were substituted into the dispersion relationships, (1)–(3), and the effective dielectric constant was computed at even-numbered frequencies to yield the solid-line curves of Fig. 2. In spite of scatter in the measurements and inaccuracies in the values of substrate dielectric constant, zero-frequency effective dielectric constant, and especially in the mode impedances, Fig. 2 shows very good agreement between the LSE model prediction and the measured values.

For the even-mode characteristics of lines 1*b* and 2*b* in Fig. 2, it is possible to find the factor *G* by using the inflection-point method described in [2], and to compare it with the value of *G* given by the empirical formula, (3). (The inflection points of the other characteristics are beyond the plotted range.) From [2],

$$\epsilon_{ei} = \tfrac{1}{4}(\epsilon_s + 3\epsilon_{e0}) \tag{4}$$

$$f_i = \frac{f_p}{\sqrt{3G}} \tag{5}$$

where ϵ_{ei} is the value of effective dielectric constant at the inflection point, which occurs at frequency f_i. Table II gives the values of *G* found by using the two methods. Because of uncertainty in the dispersion curves, the values of *G* obtained by using the inflection method are approximations dependent on the estimations of f_i at ϵ_{ei}. The significant point is that both methods give about the same values.

Manuscript received June 2, 1972; revised August 3, 1972. This short paper is based upon work performed at COMSAT Laboratories under Corporate sponsorship.
The author is with COMSAT Laboratories, Box 115, Clarksburg, Md. 20734.

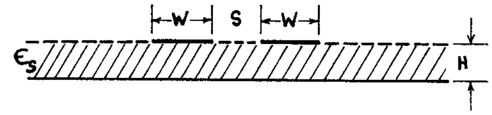

Fig. 1. Parallel-coupled microstrip.

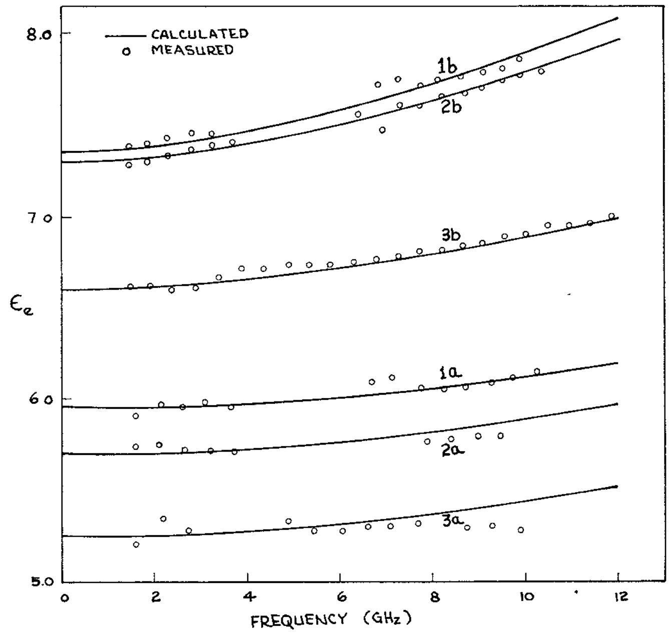

Fig. 2. Measured and calculated dispersion of coupled microstrip.

TABLE I
PARAMETERS OF MEASURED PARALLEL-COUPLED MICROSTRIP

Line	Mode	Approximate[a] Impedance	Calculated[a] ϵ_{e0}	Measured[b] ϵ_{e0}	W/H	S/H
1*a*	odd	46.8	5.95	5.95	0.86	1.12
1*b*	even	59.4	7.14	7.4		
2*a*	odd	44.0	5.80	5.7	0.80	0.69
2*b*	even	64.6	7.13	7.3		
3*a*	odd	46.6	5.54	5.25	0.30	0.19
3*b*	even	110.9	6.40	6.6		

[a] Calculated by the MSTRIP [3] program using $\epsilon_s = 10.0$.
[b] Estimated by extrapolation of measured dispersion to zero frequency.

Conclusion

The agreement between predictions and measurements has shown that the LSE-mode microstrip dispersion model and the associated factor *G*, which were derived from consideration of single microstrip